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Wilhelm Cauer and his Mathematical Device

In the latter half of the 1920s Wilhelm Cauer (1900–1945) developed his plan for an electromechanical machine to solve linear equations. By the end of the next decade he would be acquainted with four similar projects which had emerged autonomously in different parts of the world. These other machines differed from each other and from his in both concept and technological realization, but they all were capable of calculating complicated formulas in one step or in an automatically performed sequence of steps. Together they can be seen as representative elements of the pre-history of the stored-program computer. They confirm the existence of a growing demand for calculating machines which could exceed the capabilities of contemporary desk top calculators to add and multiply. They indicate that some common aspects of the scientific-technological world produced identical questions and problems on the European continent, in Great Britain and on the other side of the Atlantic, in spite of considerable differences in cultural and political situations.

In the epilogue to his excellent analysis of the situation prior to the modern electronic digital computer William Aspray considers the mosaic of what are accepted as the most important calculating instruments and machines. He then states that 'the rich connections between the technology we have set out in this book and the electronic, stored-program computer' can only be suggested, and that the 'incomplete understanding of these connections' even 'may seem odd to some readers.'[1] In that same book, Allan G. Bromley's chapter on 'Analog computing devices' presents the accepted background for the story being told here. He concludes that the listed machines 'were all of an ad-hoc nature and did not lead to any general synthesis or the emergence of a general class of machines,'[2] with the exception of Vannevar Bush's Differential Analyzer and the mechanical Gunnery Computers. Bromley doesn't mention Cauer's machine, which is appropriate since, in his view, it would be another of the broad spectrum of 'ad hoc' projects one can find scattered through contemporary journals and texts.[3]

In this essay I take another path. Cauer was not a member of some prominent community of physicists and did not achieve the kind of professional success that has drawn the attention of historians. The written sources are therefore weak, and I have to tell the story of his machine using other material. I employ a method in which I centerpiece Cauer against accounts of four comparable machines with which he was familiar. I can describe an informal connection even though I cannot verify exactly Cauer's opinions. Thus I might be able to say that he was impressed by a particular machine without knowing whether it was favorably or unfavorably. And I can infer that this knowledge of the work of others influenced him as he moved from an initial stage where he considered only mathematical, physical, and technical problems, to a second stage where he was concerned with comfortable, reliable, and semiautomatic handling, to a final stage where he appeared to worry about something like market analysis. By which time he had seen the big machine projects in the United States. So this essay can be seen as a reconstruction of Wilhelm Cauer's subjective attitude. And, although the evidence is limited, I am able to argue that his work was not completely 'ad hoc' but that it was at least partially linked to the work of others.

At the same time, I describe how problem-solving by means of mathematical models, formulae, and algorithms grew in the 1920s to become a critical difficulty, especially for programs in science and engineering. Many of the contemporary problems culminated in mathematical systems of linear equations which had to be calculated by means of the Gauß algorithm. This was generally true for science and engineering in western societies in the first half of the 20th century. Wilhelm Cauer's particular contribution was the suggestion of an electro-mechanical solution of this algorithm.

In addition, this essay is an attempt to interpret technological artefacts as well as written documents and diagrams as historical testimony. It should offer a possible historical interpretation of an artefact, by enlarging the focus to an ensemble of comparable artefacts. Unfortunately, not all machines described still exist. Those by Hull and IBM (Columbia) are preserved at the National Museum of American History. Details of the others come from written and printed material of differing quality.[4]

Wilhelm Cauer's Machine Project

Wilhelm Cauer was born in 1900 as the youngest child of a well-situated academic Berlin family. His father was the first professor for railway practice at the Technische Hochschule (TH) Charlottenburg. Many other members of the Cauer family were prominent scholars. Three of his five sisters had doctoral degrees. Wilhelm Cauer's son has termed his ancestors typical 'Bildungsbürgers,'[5] an element of society that played an important role in the German Kaiserreich.

Cauer was educated in electrical engineering, physics and mathematics at the Technische Hochschule Charlottenburg, and at the universities of Berlin and Bonn. He graduated from the TH with a diploma in Technische Physik and then enrolled at the laboratory of the Berlin telecommunication company Mix und Genest. In 1925 he returned to the TH as an assistant to Georg Hamel at the Institute of Mathematics and Mechanics.[6] This institute was the theoretical and mathematical center for all engineering disciplines at the TH.[7] At Berlin and elsewhere, mechanical statics and dynamics, particularly gas dynamics, represented the most advanced theoretical engineering discipline of the time. Still, until after the Second World War, Hamel's textbooks for mathematical mechanics played an important role in the education and working style of German engineers.[8] So, Hamel's institute can be seen as a good place for an open-minded young assistent to learn how to handle one of the great problems of mathematical engineering: how can engineers get concrete numbers from the enormous analytical formulae which were offered by mathematics and theoretical physics?

At the end of 1925 Cauer completed his doctoral dissertation 'Die Verwirklichung von Wechselstromwiderständen vorgeschriebener Frequenzabhängigkeit'. This was the first step into a field, where he would exercise great influence to the engineering of the 20th century, and where he would work until his death.[9] At Mix und Genest he had learned how to deal with patents. Cauer registered a considerable number of patents during his short life. Indeed, it seems that his first approach to the development of mathematical apparatus was to seek a patent,[10] even if he had no plan to build it.

It seems that by 1927 he already had an idea of how to solve systems of linear equations by means of an electrical device, reproducing the Gauß algorithm to transform the matrix in its triangular form. Subsequently it would be simple to calculate the unknowns with a common mechanical desk calculator.[11] Presumably he tried to obtain a patent, but there may have been problems with the application. It is not mentioned in the schedule of Cauer's patents, which was compiled after his death.[12] There is in fact only one published description of the calculating device. Cauer wrote it at the end of 1934, at the conclusion of the whole project.[13]

The crucial concept imbedded in his device represented an alternative to existing digital mechanical calculating machines and also to slide rules and cylinders. The digital numerical solution of the algorithm required a long series of calculations where the results from one were the factors for the next, and so on. So the error inherent in rounding numbers, and in multiplying and dividing rounded numbers, grew with the number of the equations. Cauer's concept incorporated an analogue device, the precision of which, he believed would be sufficient for most engineering problems. The critical element was an electrical Wheatstone bridge with variable decimal resistors. The bridge circuit represented the analog part, and the decimal resistors the digital part of this hybrid device.[14]

In Cauer's apparatus the coefficients were represented by resistors. The calculation proceeded in a step-by-step elimination of the equation coefficients by balancing the different bridge circuits. So it was not necessary to measure the currents absolutely. The system of linear equations

$$a_{11 \times 1} + a_{12 \times 2} + a_{13 \times 3} + a_{14} = 0$$
$$a_{21 \times 1} + a_{22 \times 2} + a_{23 \times 3} + a_{24} = 0$$
$$a_{31 \times 1} + a_{32 \times 2} + a_{33 \times 3} + a_{34} = 0$$

could be transformed into the triangular system of these equations.

$$a_{11 \times 1} + a_{12 \times 2} + a_{13 \times 3} + a_{14} = 0$$
$$a'_{22 \times 2} + a'_{23 \times 3} + a'_{24} = 0$$
$$a'_{33 \times 3} + a'_{34} = 0$$

First the resistors a_{11}, a_{12}, a_{13}, a_{14} were set as the coefficients of the first equation. The switches U changed the sign. In the diagram parallel connections symbolize the positive sign, crossing connections the negative ones. Now the upper resistor row is set with the coefficients of the second equation. The current I_1 through the circuit is tuned by changing R_1 so that the voltage sum $a_{21}I_0 + a_{11}I_1$ becomes 0. This state occurs if the galvanometer G indicates no current. Then in the next row of resistors a'_{22}, a'_{23}, a'_{24} is set so that the current is constant, and a'_{22} is set by changing R_2 so that the voltage sum $a_{22}I_0 + a_{12}I_1 1 + a'_{22}I_2$ is 0. The galvanometer G again indicate no current when S is set to 2. And so on.

Cauer stated that he had calculated several complete systems using this method at Professor Max Reich's Institut für angewandte Elektrizität at the University of Göttingen.

Cauer began constructing an automatic version of his machine for three equations with three unknowns during the first half of 1930, but he could not complete it because of lack of funds. We know Cauer's circuit design

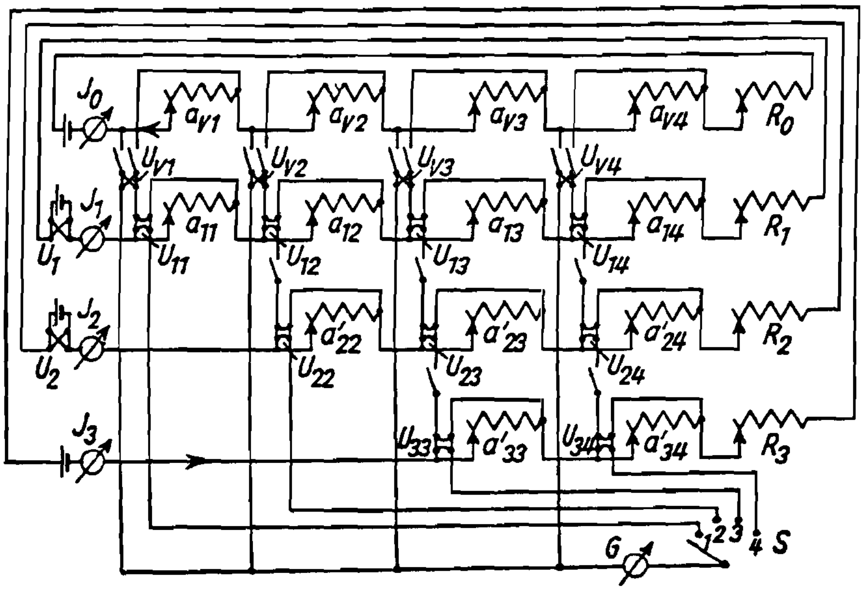
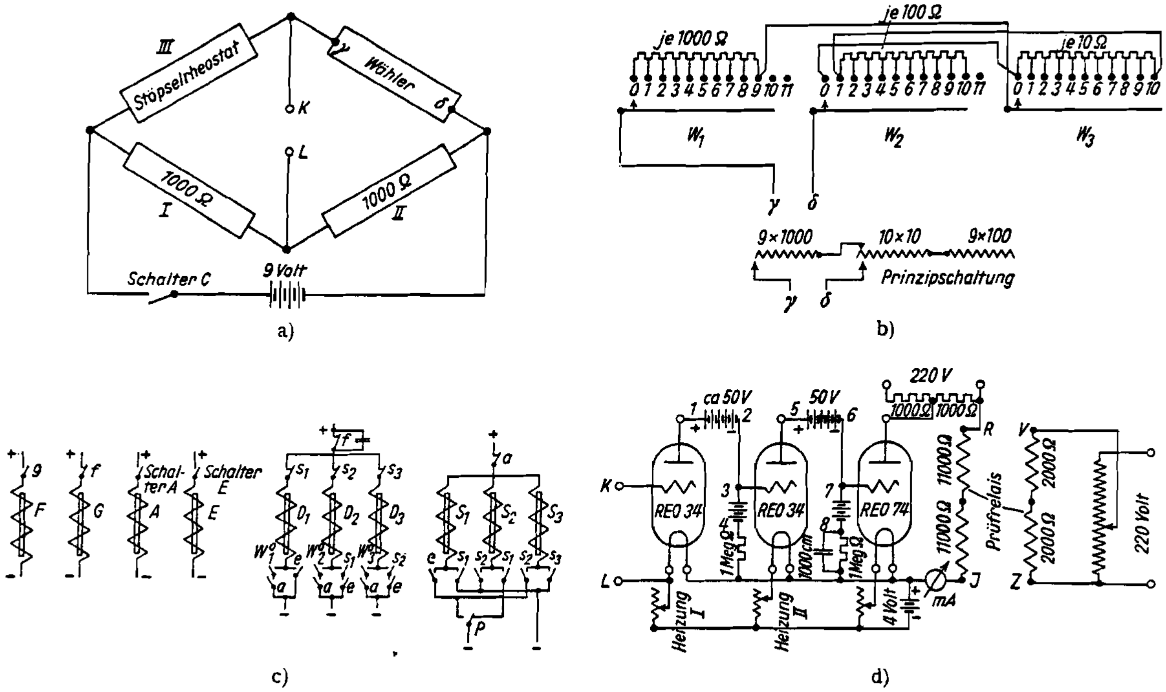


Figure 1. Elementary diagram of the machine, as it was published by Cauer in 1935 (Elektrische Maschinen *... p. 150). Notice the diagrammatic approach is highly standardized, but in a way that provides an abstract picture of the technical artifact which makes aspects visible one cannot see directly. However, engineering diagrams like this cannot elucidate the differences among the machines discussed here.*



a)

b)

c)

d)

Figure 2. Diagram of the components of the automatic version:
a) Wheatstone bridge circuit. b) four resistance arms of the bridge (three fixed and one variable), c) magnets and switches controlling the variable resistance),
d) dc amplifier with polarized relay.
(Cauer, Elektrische Maschinen…, *p. 150)*

from his publication: the coefficients were set by means of decadic resistors with 1000, 100, and 10 Ohms. Cauer wrote that a machine for ten equations would be feasible under the same principle and that it would be as expensive as Mallock's machine (below). To make it automatic, the galvanometer was replaced by a polarized relay connected with a d.c.-voltage tube amplifier. In the bridge diagram (a) resistors I and II are fixed. Resistor III is measured by reading the decade resistor gd, which is automatically set by three rotary switches W_1, W_2 W_3. The automatic setting of one coefficient took 3 seconds, Cauer reported.

Cauer emphasized that the elimination procedure could be arranged so that the capacity of the machine would be never exceeded, and also that from the calculated coefficients the value of the determinant and the errors could be evaluated. But this could not be done automatically, and the solution of a complete system of linear equations had to be complimented by separate manual exercises.

At that time Cauer was a Privatdozent for Angewandte Mathematik at the mathematical institute of the University of Göttingen. With his habilation thesis completed in 1928, he had had the opportunity to obtain an assistantship. This had been created under an agreement between the Rockefeller Foundation and the Prussian authorities, by which the famous mathematical institute of David Hilbert and Felix Klein got a new building and research positions. In the same year Richard Courant was named director of the institute. From Constance Reid's biography of

Figure 3. Photograph of the machine, probably in 1930. Allegedly the picture shows it in the entrance hall at the Mathematical Institute before Cauer left for the USA. He may have taken the picture to show to his American sponsors.
(Cauer Papers)

Courant we get an impression of what a complicated man he was, and also how he managed the institute.[15] Nonetheless in 1928 Cauer had been able to push his Habilitation, for the subject Angewandte Mathematik.

Cauer used Courant's good relations with the Rockefeller Foundation to get a fellowship for one year, starting in September 1930, at Vannevar Bush's institute at the Massachusetts Institute of Technology (MIT). The correspondence with Bush just before Cauer's departure includes a discussion of the mathematical device. In one letter Bush mentions that he had discussed the subject with Norbert Wiener, but ultimately he was skeptical of Cauer's plans.[16] In Cauer's official report of the stay the calculating machine project is not mentioned. Nevertheless one can read that he had inspected several existing machines at MIT and at other institutions in the USA.[17]

How well Cauer and his wife mixed with others at MIT is unclear. Norbert Wiener, in his memoirs, mentions them only once.[18] But Cauer certainly learned that problems were handled differently in America. During the thirties Bush and his colleagues planned and built a whole series of important calculating machines. Bush, who was international recognized, broke through narrow academic rules in a way that was not possible in Germany, and he still had not reached the pinnacle of his career. In contrast, three years after his return to Germany, Cauer had to stop his project without any chance of resumption. This was due at least in part to the fact that the new telecommunications theories had not found an adequate place at the Technische Hochschule or at the universities. Academic institutions did not welcome a mathematical working engineer like Cauer. As a consequence, his academic career ran aground and he had to take up work as an engineer in an industrial laboratory and his influence on academical theories of engineering was modest.[19] When in 1941 the first volume of his major work 'Theorie der linearen Wechselstromschaltungen'[20] was published, he still held the position of head of Mix und Genest's research laboratory and gave lectures at the Technische Hochschule.

Cauer's difficulties in achieving an academic appointment at the university were also political. His best chance came in 1933. His fellow teacher at Göttingen, the statistics professor Felix Bernstein, was in the United States when Hitler came to power and he decided not to return. Cauer, who was married and had two children, applied for the vacant position. He was unsucessful, apparently because he was not a member of the Nazi party and he wasn't supported by the strong Nazi group at the university.[21]

Frustrated in his pursuit of an academic career, at the end of 1933 Cauer wrote letters to 18 scientific institutions and industrial companies, inquiring into their needs for solving linear equations. He posed seven questions, including the number of such problems which were dealt with in a year, the time needed for them, the number of unknowns and figures, and the type of instruments used. Most of the companies were concerned with static problems.[22] The solution of a system with 4 or 5 unknowns took anywhere from 45 minutes to 4 hours, 6 to 10 unknowns could take from 6 to 20 hours. An airship company (probably Zeppelin) had dealt with problems up to 24 unknowns and was interested in systems with 38 unknowns. There were photogrammetry problems with 68 unknowns, and an astronomical institute had no less than 360. The required precision 'in the engineering cases', as Cauer expressed it, would be no more than one per thousand. The demand would grow if one could calculate faster, thought Cauer.

In 1934–35 Cauer abandoned not only his plans for a calculating office and the completion of his mathematical machine. He left the university and joined Fieseler Aircraft at Kassel not far from Göttingen. In 1936 he was given a position as head of the research laboratory at Mix und Genest, and the family returned to Berlin.

Clark Leonard Hull's Machine

Cauer was particularly impressed by the design and the application possibilities of the calculating machine of the American psychologist Clark Leonard Hull (1884–1952). This machine was nearly unknown in

Germany and Europe. The fact that a psychologist built a self-designed semi-automatic calculating machine challenges the widespread impression that only people in commerce and engineering designed calculators. Some decades later experimental psychology would become an important part of the computer-using community. Nevertheless a career like Hull's, which illustrates a very pragmatic attitude, can hardly be imagined in Europe. It is also hard to imagine a greater contrast between the self-made scientist Hull and that of the academically-trained Cauer. These differences also found expression in their machines.

Born on a farm in New York state Hull attended a one-room school and later passed a teacher's examination. Because of illness he had to give up his planned career as a mining engineer and decided to become a psychologist. In 1914 he became a teaching assistant at the University of Wisconsin and wrote a doctoral dissertation on Chinese ideographs, which was soon accepted as a standard work on learning theory. In 1918 he was appointed instructor and was given responsibility for the course in experimental psychology.[23] Here he took over the work on tests and measurements in the Psychology Department and became interested in aptitude tests.[24] The interpretation of the tests depended on an extensive use of correlation coefficients, which the limited capacity of the then-available calculators made a tedious and error-prone process. So in February of 1921 he began to design a special machine to do the job automatically. He developed mechanical reproductions of the formulas of Thurstone for the Pearson coefficient of correlation and of Rumel for standard deviation.[25]

Hull planned all of the processing and mechanical details. In April 1923 the construction of the machine was commenced and it was presented, in a provisional state, to the Madison meeting of the American Psychological Association in December that same year. It was sufficiently perfected in the late summer 1924 to solve multiple regression equations automatically as well as to do practical correlation work on a large scale, with about 150 coefficients being computed. In December 1924 he demonstrated the machine before the Washington meeting of the American Psychological Association both as an automatic calculator of multiple regression equations and as an automatic correlation calculating machine. From April until December 1925 it computed over a thousand correlation coefficients and standard deviations.

In 1923 Hull had considered 'as a somewhat Utopian speculation' that some day a computing apparatus might be constructed which would solve multiple regression equations automatically, and thus yield mechanically, and cheaply, series of aptitude forecasts for vocational guidance.[26] But soon after, while working on the final details of the machine's design, which was planned only as an automatic correlation calculator, he hit upon the basic principles of such an aptitude-forcasting device. He found similarities of principles between the two machines such that the correlation machine as it

Figure 4. A sample strip of perforated tape (10 cm wide). Holes within the rectangles represent individual digits. Thus the first number is 9, the second 24, the third 365, the fourth zero. Holes at the left edge of the box for the first digit of each number control shifting from one number to the next. Successive columns of data to be correlated follow one after the other on the tape, a single blank space being interposed between each. (Hull, 'An Automatic Correlation Calculating Machine,' p. 526. The same picture was published by Hull in Aptitude Testing, *p. 488.)*

had been finally constructed served both purposes equally well. It could be used in two modes: one for the calculation of the arithmetic mean, the standard deviation and the coefficient of correlation; and one for the continuous calculation of the multiple regression equations, especially for making wholesale multiple aptitude predictions for purposes of vocational guidance and selection. No question that such a seemingly flexible machine was interesting also for Cauer's problems. Particularly promising, while the machine was doing its calculations, the attention of the operator apparently was not required at all. On several occasions, as Hull reported, he himself had started the machine on a long and difficult column of computations, then he had locked the laboratory and gone out to lunch.

The electrically-driven machine was essentially a products-sum calculator which rendered any special correlation tables or preliminiary hand computations superfluous. It calculated automatically the values required by the formulae, one after the other. The formulae were then ready for

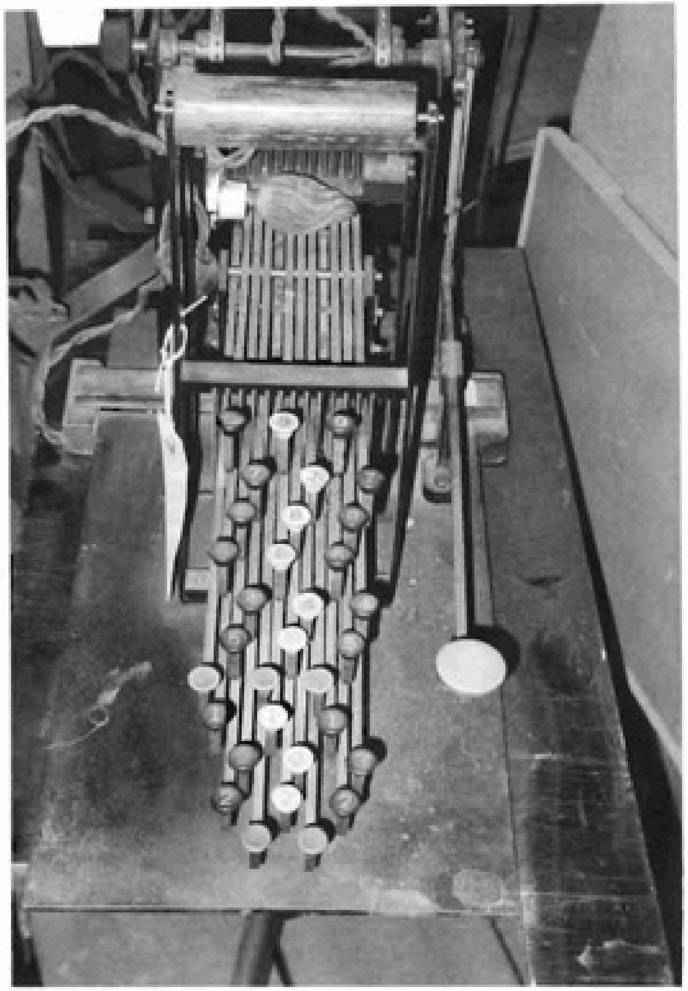


Figure 5. The special punch machine without tape. Notice the toothed wheels for moving the perforated tape, the plates to contol the punched numbers, and the special bulb which is able to provide a diffuse light for all positions of the plate. (Photo taken by H. Petzold in the NMAH, 1997.)

the final solution. The coded data then were punched into a strip of tough kraft paper, 0.07 inches thick an 4 inches wide, which was perforated on the edges somewhat like movie film. These edge perforations engaged the teeth of spockets on recorder and calculator in turn, insuring the precise movement of the paper through each. Numbers up to 999 could be recorded. A Veeder counter showed automatically the number of the item recorded at any given stage of the process. Cauer's equivalent solution with a rotary switch seems smarter. But Hull's punched tape also stored the data which was not possible with Cauer's device. Since the same regression equation was likely to be used over and over again the data for it were punched into a more permanent material such as a thin metal band, with 24 numbers on one foot of tape. The successive columns of data were correlated following each other on the tape, a single blank space being interposed between each.

Hull also considered how to correct errors. He placed a transparent number plate on the recorder over which the paper record was drawn as it feeded through the apparatus. The number corresponding to each perforation could be seen distinctly through the perforation itself. If an error was found, a small square of gummed parcel-wrapping tape could be pasted over the faulty perforation and the correct hole was repunched by the recorder.

The machine was built on a steel table, 26 inches by 32 inches. It was driven by a 1/4 HP electrical motor. Both at the multiplier and multiplicand there were steel fingers arranged in a pattern, one finger for each possible digit. These fingers periodically descended upon the perforated data strip. Those fingers which fell upon perforations passed through and thus directed the action of the machine very much as the hands of an operator might do by operating keys. When a multiplication had been completed the strips were moved to the next number. Squaring operations were performed simply by running duplicate data strips through the machine in parallel, one through the multiplier and the other one through the multiplicand. The action of the machine was purely mechanical, electricity being used only to drive the motor.

All that the operator was required to do was place the strip containing the numbers of the first column in one position, the strip containing the numbers of the second column in the other position and then press the starter. The machine then, automatically multiplied each pair of numbers one after the other continuously, adding up the products as it went along. When the machine reached the bottom of the columns, whatever their length, it stopped automatically and *(A × B) could be read from a dial.

Part of the funds for the machine came from the University of Wisconsin, a larger part from the Committee on Scientific Problems of Human Migration of the National Research Council. The machine was built by the mechanic Harold C. Kidder from the university in cooperation with the Chief Mechanician, O. E. Romare.

By December 1925 two machines had been built, the first one for the Wisconsin psychological laboratory, the second, an improved model, for the National Research Council. Hull then offered replicas of the calculator and the two auxiliary machines for about $ 1200.[27] As Hull reported, the establishment of a central correlation bureau had been suggested, since many institutions needing correlation work did not have enough work to make the purchase of a machine worthwhile. At the end of 1925 Hull wrote 'It seems likely that such a bureau provided with one or two of the machines ... together with other modern statistical aids and a specially trained person, could do a good share of the correlation work of the United States and with a promptitude, economy and accuracy previously unknown.'[28] Today one machine is situated in the Smithsonian's National Museum of American History.[29]

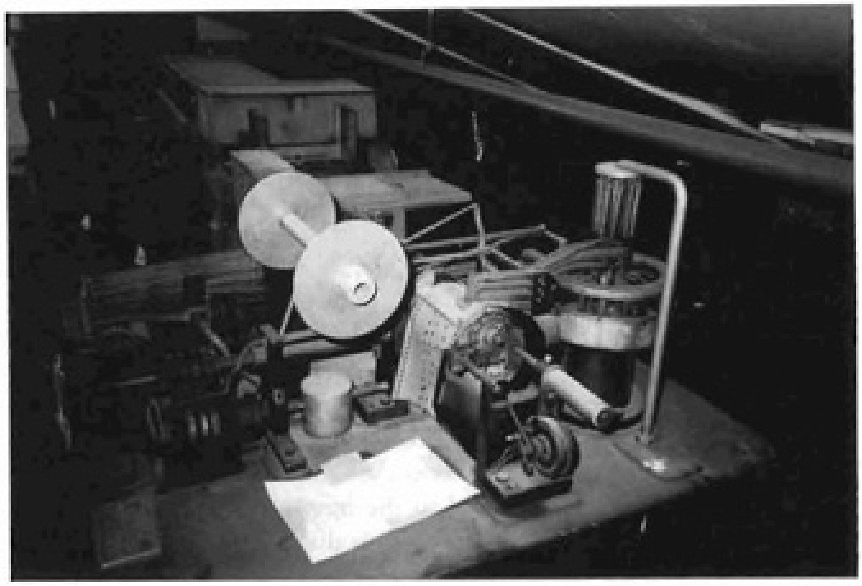


Figure 6. The calculating machine in the depository of the NMAH. (Photo taken by H. Petzold, 1997.)

Hull actively promoted the machine.[30] In a textbook on aptitude testing, published in 1928, he described the rapid development of psychological testing during the previous years.[31] He was obviously proud that the former primitive methods had been replaced by scientific ones. Naive approaches to relationships between test and aptitude, gave way to better theories and clearer expectations. 'In a word, aptitude testing, like medicine and engineering, is ceasing to be a job for amateurs and is becoming the work of technically trained professionals.' (V). All of which was made possible by the existence of the machine. This is exactly the kind of argument that would have appealed to Cauer: his machine should in like fashion provide a new basis for the whole of mathematical engineering.

In 1925 Hull had argued in several papers as a practitioner: his machine would eliminate the drudgery and the persistent arithmetical errors from calculations of standard deviations and Pearson product-moment coefficients of correlation. In his 1928 book the arguments were more fundamental. He referred to Plato's argument in the *Republic* that everybody should be given his place in the state, and particularly in the army, in accordance with his abilities. To do so, it was necessary to detect those abilities. Plato's utopian dream, 'each man work at a single occupation in accordance with his natural gifts' was not possible without psychological tests.[32] Even if Cauer had comparable ideas he would not have dared publish them.

Hull saw the development of his calculator as 'another system of making aptitude predictions from forecasting formulae' and as an integral part of a comprehensive program of vocational education, which he

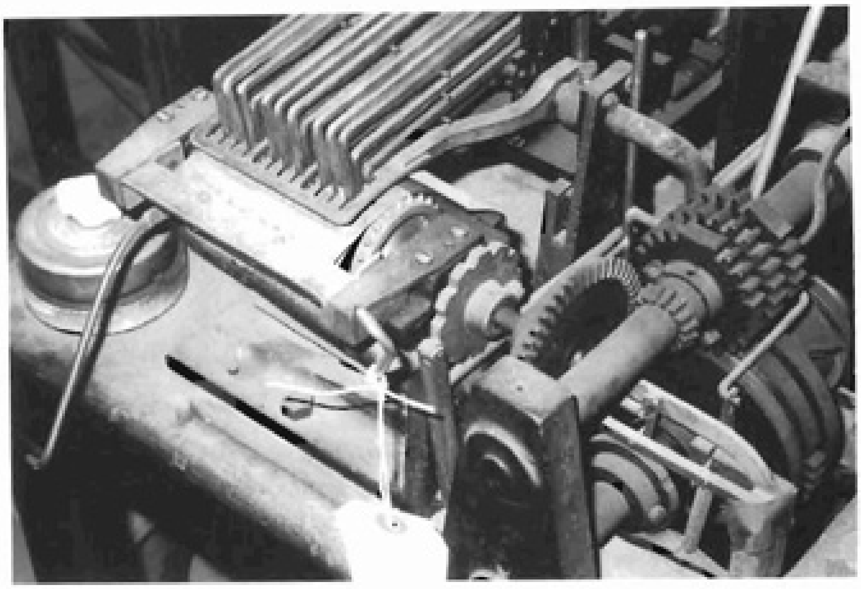


Figure 7. The detail shows arrangement of the reading fingers. One can see the massive design of the parts of the machine, which is stronger than any recent calculating machines. (Photo taken by H. Petzold, 1997.)

sketched for the first time in 1923. He called for the construction of one universal battery of tests which would sample all important aptitudes. The battery would cover some 30 or 40 different elements and its execution should take one day or more. Based on this battery separate formulae could be constructed for prediction. Forty or fifty different equations would have to be solved. All in all there would have to be about 1500 multiplications where the products had to be added. In cases like this only the machine offered freedom from arithmetical errors and other mistakes.[33]

Undoubtedly Cauer was impressed by Hull's sophisticated and obviously successful machine, especially since success in this case meant significant academic acknowledgment. He hoped to get a Hull machine to Courant's institute, and on 12 July 1929 he wrote to Hull in Wisconsin saying that he had read Hull's 1925 article in the Journal of the American Statistical Association 'mit großem Interesse', and that he would suppose that Hull's machine could be used 'auch zur automatischen Bewältigung mancher anderer zeitraubender Rechnungen, wie z.B. die Herstellung der Normalgleichungen bei Ausgleichsrechnungen … Doch wäre es dafür erwünscht, eine Maschine mit größerer Stellenzahl zu besitzen.' He continued 'Wegen der außerordentlichen Wichtigkeit einer derartigen Maschine für numerische Rechnungen besteht der Plan, für das Göttinger Mathematische Institut eine solche Maschine anzuschaffen oder zu bauen' and 'falls sich der Preis tatsächlich in den von Ihnen angegebenen mäßigen Grenzen bewegt'. He apparently got no answer, but he clearly continued to be interested. He was responsible for a footnote mentioning Hull's machine which appeared in a publication by the mathematician Theodor Zech in March 1929.[34] And in a letter dated 3 January 1930 Cauer asked Bush if he could say anything about the usefulness of Hull's machine.[35] During his stay in the United States Cauer met Hull and saw the machine at the Institute of Human Relations at the Yale University. Unfortunately there is neither a record of his impressions at the time nor an indication in the Cauer papers if he made any further effort to get a Hull machine to Göttingen.

The 'Columbia Machine' of IBM

Cauer was familiar with the punched card system. He inspired Theodor Zech to consider the use of this system for harmonic analysis. Undoubtedly it was also Cauer who inspired Zech to collect data associated with renting and operating a machine. The data came from the Vulkan dockyard in Hamburg and from the Dehomag company.

During his stay in the USA, Cauer saw the new tabulating machine which had recently been developed at IBM. At this time IBM had been

Figure 8. The Columbia Machine—with Benjamin Wood at his calculation center in 1930 (Archives of the NMAH)

Figure 9 The same machine in 1997 withPeggy Kidwell at the Smithsonian. (Photo taken by H. Petzold, 1977)

building and marketing punched-card machines for three decades. In retrospect there is no question that the development and the success of the punched card system by IBM and some other smaller companies paved one of the different ways that would lead to the computer. The first president of the company and creator of the name IBM, Thomas J. Watson, executed the strategical switches. Some of them can be designated as historic not only for the company but generally for the development of information technology. One turning point in the history of the company occurred at the same time that Cauer was planning his equation solver in Germany.

In 1928 army psychologist Benjamin Dekalbe Wood became a professor of Collegiate Educational Research at Columbia University. Son of an itinerant cattle rancher in Brownsville, Texas, Wood, like Hull but unlike Cauer, did not grow up in an academic environment. His school education had been short and he was in most fields self-taught. As a student at the University of Texas with no high school credits, he had studied Plato and Aristotle with an obviously naive and irreverent spontaneity, and was impressed by Malthus.[36]

The chronicler of IBM and biographer of Thomas J. Watson, William Rodgers, tells us that Wood left university with a Batchelor's degree convinced that, in the words of H. G. Wells, 'civilization is a race between education and catastrophe' and was determined to intervene. After his military service, he attended Columbia University and for his doctoral dissertation applied E. L. Thorndyke's thesis that 'whatever exists at all, exists in some amount' to the measure of human intelligence. As an assistant to the director of the Columbia College, Herbert Edwin Hawkes, he advised the students and found himself confronted with the question of how their future development could be assessed during the term of their vocational education. His methods of tests and valuations attracted some attention. Wood was also in contact with John Dewey and his Progressive Education Association.

With Hawkes' support Wood got substantial funds from the Carnegie Foundation,[37] the Commonwealth Fund, and the Educational Board. His investigations also made the authorities of the education bureauctacy listen attentively. When the interpretation of test results exceeded the capacity of current processing, Wood wrote to the chief executives of ten corporations in the equipment business and asked for support. It is part of the IBM legend that its autocratic president was the only one who answered, and that Wood convinced him in a long dialogue, after which Watson supported Wood with a calculation center based on the punched card system.

Alledgedly Wood convinced Watson that the limits were defined by the productivity of the machines. The goal was to work with the speed of light. As Rodgers reports, Watson put pressure to his technical staff to achieve this goal. He rejected their arguments that the possibilities of technology were limited.[38] Even if the whole story, written forty years later, was fabricated as propaganda, there is no doubt that during the following decades Watson was determined to open up new markets in the sciences through new modifications to the standard machines.[39] Watson knew that punched card technology would go far beyond the hitherto-existing markets of IBM, indeed it had no limits at all. This attitude is in stark contrast to that which surrounded Cauer, or even Hull and Wood.

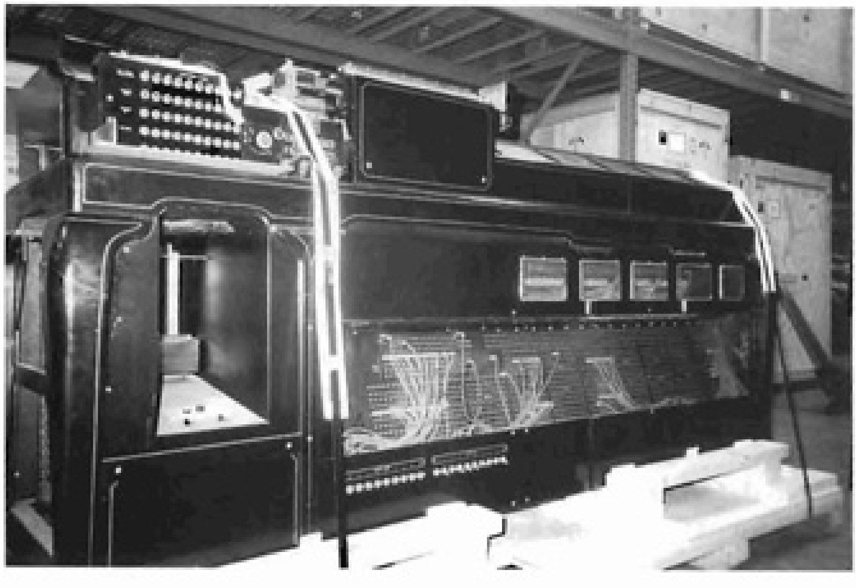


Figure 10. The front of the Columbia Machine is unique not only in the way the input-output technology is treated, but also in the typical prewar IBM design: completely black with golden ornamental lines. (Photo taken by H. Petzold, 1997.)

In 1929 the IBM president installed a calculation center with punched card machines for Wood at Columbia University. From Watson's point of view it would play the role of a model and a playground, where protagonists' activities and the results they achieved could be observed. The first important modification of a standard machine, incorporating the suggestions of the scientists around Wood, was the so called Columbia tabulating machine. It was a modification and enlargement of then new type IV tabulating machine. Engineers James W. Bryce, George F. Daly and Gunne Lowkrantz in the IBM plant at Endicott, New York, built a machine with two counters for transfer cycles and ten counters with ten figures each. The machine had the ability to transfer numbers between the different counters. However this still was far from being a computer; the machine could only add. IBM marked the new claim with a patent where the text particularly mentioned scientific applications.

This machine arrived at Wood's calculating office in December 1929 and is known as the 'Columbia Machine' or 'Ben Wood's Machine.'[40] It remained unique and has also found its place in the National Museum of American History.[41] The 'Columbia Machine' was IBM's first step in a line of development which lead in 1935 to its commercial test scoring machine, the IBM 805. In Germany the Dehomag developed at the same time the tabulator machine D11 without any contacts to scientific institutions or individuals.[42]

I am convinced that this was the 'new machine' which Cauer saw during his visit and is mentioned in his report. But we do not have any more direct information about his thoughts. He must have been impressed by IBM's interest in applications of the punched card system for scientific calculations, particularly correlation calculations, and also by IBM's readiness to modify the conventional machine for a new scientific market. But he must also have noticed the quantities of data which were needed for these sociological and statistical evaluations, which exceeded the quantity of the coefficients of Cauer's equations by many times. He must also have recognized the particular limitations of the punched card system. Most impressive to Cauer would have been the high standard of reliability which was set by the IBM devices, and which was impossible to reach with his own machine in the contemporary Göttingen situation.

It is interesting to speculate what had been happened if Cauer had been the one to approach Watson with his method of solving linear equations. Chances are that he would not have been as successful. Partly it would have been a personality problem: Cauer did not have the self assurance possessed by Wood. But Cauer also would not have had the vision of a market with hundreds and thousands of equations where millions of punched cards would be needed—and where the small errors allowed by his system would have become intolerable.

J. B. Wilbur's Simultaneous Calculator

In the spring of 1935 Cauer became aware that at MIT J. B. Wilbur was building a machine that could solve a system of nine equations with nine unknowns, working on a 'purely mechanical-kinematical principle'. It functioned on what Bush had described as a more precise version of the well-known principle developed fifty years before by Lord Kelvin.[43,44] Wilbur did not claim to be the inventor of the machine; he described his role in the project as a 'clearing house for the ideas of those who work with him.' Obviously there had been many such ideas. He thanked Bush who was then the Vice President and Dean of Engineering at MIT. The technical realization of the machine was made possible by the Singer Sewing Machine Company and its President Sir Douglas Alexander. The head of the project was Professor Charles B. Breed, Head of the Civil Engineering Department at MIT.

Certainly Cauer had not forgotten his exchange of ideas with Bush from 1930/31 when he revealed the details of his own concept. Now he had to consider the Wilbur equation-solver, based on another principle, which was not new and which Cauer had rejected six years before.

Wilbur's published description of the machine was written in December 1936, too soon to include a critical report on its practical use. It leaves open the question of why they had decided on Kelvin's principle (an approach that Cauer had explicitly rejected), which had some problems and inevitably made the machine unwieldy in size. In 1934 a rough prototype had been sufficiently successful in solving systems of linear equations with real coefficients so that a decision was made to build a larger machine. This was essentially complete by 1935, and in 1936 it was used for solving systems of nine equations. Wilbur emphasized that it could be also used for calculations of the unknowns of more equations if some restrictions on the form could be made. He reported that 'theoretically' this could be done without restrictions with modified versions of the machine.

The optimistic generality of Wilbur's statements can be contrasted to the skepticism expressed by Bush in his letter to Cauer in 1930. In 1936 Wilbur wrote confidently that the machines would be helpful for the progress of technology and research.[45] But apparently it was characteristic of the times that any real market analyses would be left to the financiers and was not a concern of the scientist-inventors.

Wilbur's machine consisted of a heavy steel case with ten steel plates arranged to swivel inside. Nine plates corresponded to the nine unknowns, the tenth to the constants. The solution of nine equations with nine unknowns, to a precision of three digits, took between one and three hours. Although this was better than working with a desk calculator (where calculation of eight equations with eight unknowns took on the order of eight hours) Wilbur hoped that with practice the time could be shortened. For precision problems Wilbur mentioned only that the time

Figure 11. Wilbur sitting before the Simultaneous Equation Solver, setting a coefficient. (Photo MIT Museum)

taken depended on the type of the equations. He believed that for ordinary systems of equations the error would not exceed 1% of the largest unknown. In most cases precision would be greater. The machine was suited for stepwise approximation, which made possible precision to any degree desired. Under favorable conditions, a system with 18 equations had been solved in 7 or 8 hours with a precision of four or five digits. The same calculation done with a conventional desk calculator needed 32 hours. A greater number of unknowns would increase the time saved significantly.

Wilbur wrote that research and development with this type of mechanism should be continued, stating confidently that a machine for the direct solution of a greater number of equations was possible. However, even on the first machine, small modifications were necessary. One technologically weak point, which Wilbur already had noticed, was the type of steel tape used which was expected to lose its flexibility and tension. The technical skill required to handle the machine needed to be improved. And he expected reading accuracy to be increased. In an improved bigger machine time could be saved by development of an automatic frame for setting coefficients and constants (another indication of the move towards automation).

The only practical use of Wilbur's machine was by the prominent Harvard economist Wassily W. Leontief from Harvard University.[46] Leontief's original calculation of a national economic 42-sector input-output model required about 30 million multiplications. He simplified the data into a 10-sector grid, but even that would have required 450,000 multiplications or, as he reckoned, two years at 120 multiplications per hour. Leontief recalled in an interview in 1969, 'You could really change the coefficients slightly by simply sitting on the frames, and if they did not give too much this meant that the solution was relatively stable.'[47]

Wilbur would never build an improved machine. He decided to make his career as a professor of civil engineering at MIT and not as a designer of mathematical machines.[48]

Mallock's Machine

Our last machine is that of R. R. M. Mallock in England. Both in form and function it resembled Cauer's more than any other. Its story suggests a similarity of approaches between Germany and England (more so than with the United States) and also how Cauer's machine might have evolved. The emergence of Mallock's machine, and a new publication by Bush,[49] gave Cauer reason to believe that his ideas had been confirmed, and he wrote a paper claiming priority.

Rawlyn Richard Maconchy Mallock was born in 1885 in South England where his father represented the community of Torquay in Parliament. Mallock took an engineering apprenticeship in Manchester

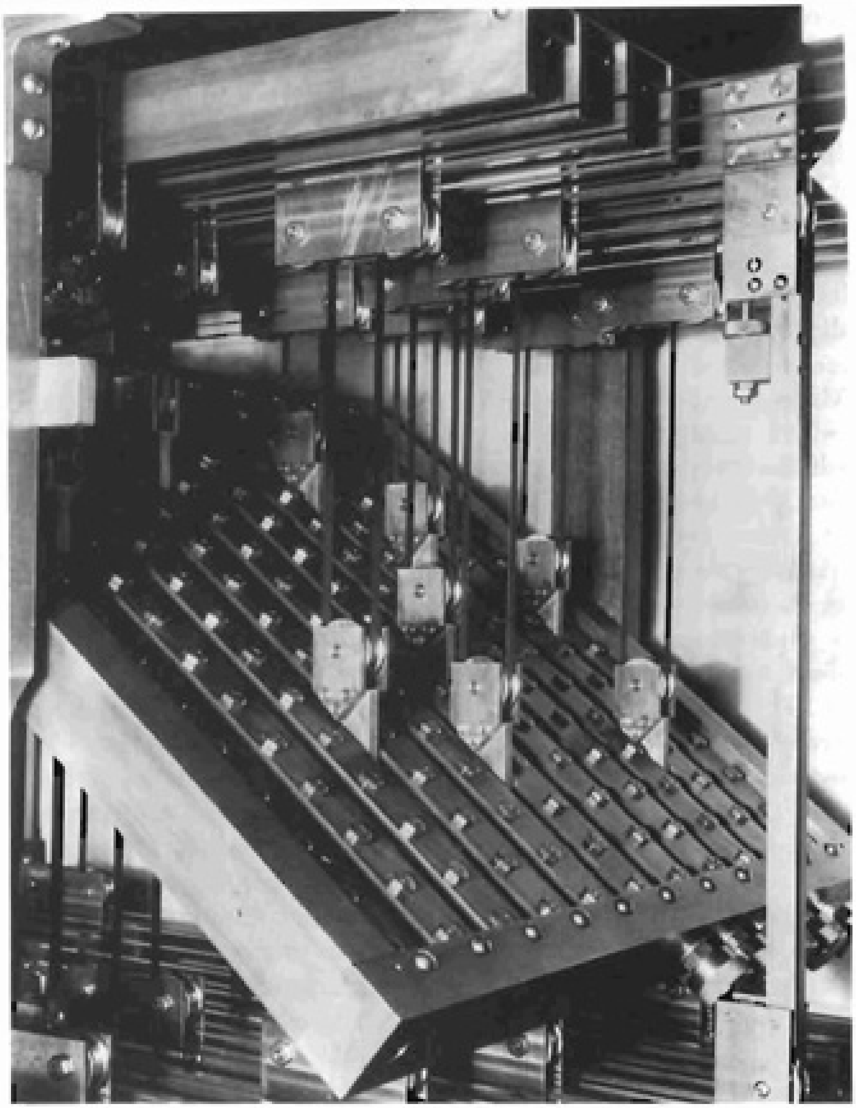


Figure 12. The detail shows one of the plates with some of its nearly 1000 pulleys leading the steel tape. Above and below some of them can be shifted horizontally. Notice the controlling knobs at the plate. (Photo MIT Museum)

in 1906, and 1908, after he had completed part one of the mathematical and mechanical sciences tripos at Trinity College, Cambridge. He went to Canada for some time and satisfied his war service with the armaments company Armstrong Whithworth. After the First World War he was an electrical expert at HMS Vernon, a research institution of the British Admiralty. At the end of the 1920s he returned to Cambridge and worked there until his retirement in 1937. He died in 1959.[50]

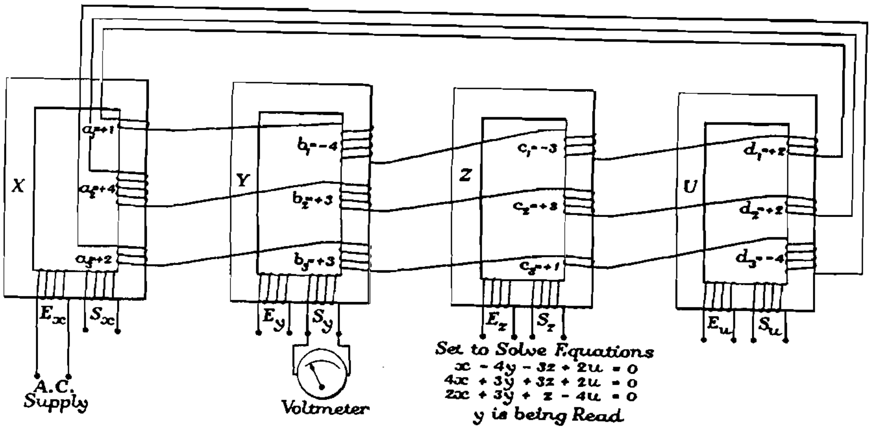
Mallock gave the same reasons for his project as Cauer and the others: 'In connection with many problems of engineering and physics it is necessary to solve sets of linear algebraic simultaneous equations involving a large number of unknowns; for instance in the determination of secondary stresses in bridges and other structures sets of equations involving from ten to twenty, or even more, unknowns may occur and the labour involved in the solution when the number of unknowns is more than about six is very great.'[51]

In 1931, almost at the same time as Cauer, Mallock constructed his prototype, which solved systems with 6 linear equations with a mean error of 0.4%. The mean error had been reduced to 0.1% in the following machine, built in 1933.[52] In Mallock's eyes this was enough for the solution of engineering problems. Like Cauer, Mallock designed two additional frames which could set the rotary switches automatically by means of several relay circuits.

Mallock made a contract with the Cambridge Instrument Company, to which he granted his patent rights for the construction of future machines. Charles Darwin, the chairman of the Company and later head of the National Physical Laboratory in Teddington, mentioned the contract in his report for the Royal Society, 'in an endeavour to create a real market for this machine'[53] At the beginning Mallock contributed £200, with the remaining cost of construction to be payed by the company. The expected profit was to be divided according to this ratio until Mallock had received 400 Pounds.[54]

In 1933 Mallock directed construction of a machine at the Cambridge Instrument Company. After its completion it was taken to the Cambridge University Engineering Laboratories, where Mallock was employed. Later, in 1937, it was bought by the university's Mathematical Laboratory (for £1,750; Croarken is convinced that the company lost on the deal, all things considered) where one of its users was the computer pioneer Maurice Wilkes.[55] In 1933, Darwin presented the machine to the Royal Society, and published a comprehensive report in its *Proceedings*.[56] A further report was published in the widely-read technical journal *Engineering*.[57] In addition, professional circles were informed of the

Figure 13. The principal diagram shows the relationship with Cauer's design and is cited in his publication. Where Cauer used an electrical circuit Mallock took four magnetic 'compensators.' (Engineering, *1934, p. 698)*



machine at the Royal Society Conversazione in May 1933. All of which illustrates that Mallock had the kind of support that Cauer lacked.

As a consequence, the Mallock machine commanded a certain degree of interest for about a decade. But eclipsing both Mallock and Cauer was Bush, who described (and promoted) his Differential Analyzer in 1934 at the International Congress of Applied Mechanics in Cambridge, England.[58] Quite in contrast to Cauer's stay in United States, which was noticed only by ten or twenty fellows, Bush's appearance was a high-profile affair which can be seen as part of a new scientific-political initiative on the part of the United States, supported (as Cauer's trip had been) by the Rockefeller Foundation. Over the succeeding years differential analyzers—often called 'Bush machines'—were constructed in several European countries, this putting the stamp of mechanical analogue technology on mathematical machines until the end of the 1940s.

There were reasons for the different reactions of the scientific and mathematical communities to the machines of Mallock and Bush. For one thing, the Differential Analyzer had had several forerunners both in terms of machines and practice ranging back to the beginning of the 1920s. Moreover, it could solve differential equations, giving it a clear advantage over a machine that could only solve less spectacular linear equations.

The different ways of working may also have played a part: The Differential Analyzer, as an analog machine, was fed with curves, was operated with curves and produced curves. And most scientists and all engineers were taught to visualize their problems in terms of curves. Mallock's machine (like Cauer's) had a digital link to the omnipresent classical, industry-made desk calculators. But to a certain extent it lost this advantage because of its hybrid nature; one could say that its results were measured rather than calculated or reckoned.

Mary Croarken, who has done extensive research on Mallock and his machine, states that the prominent figure of Comrie was never particularly interested in Mallock's machine and that he never studied it intensively. The reports in *Nature*[59] and in other journals confined themselves to descriptions of the technical function and failed to evaluate its reliability or its relevance to the scientific and engineering community. Furthermore, Mallock, like Cauer, does not seem to have had a prophet's temperament.

Mallock's machine is the only one of this group which was used—not intensively, but several times—and where the experiences of that use are known. As the mathematician A. C. Aitken reported in *Nature*,[60] the setting of the switches and the plugging of the wires for the calculation of a system with 6 equations, took half an hour. When a solution had been determined it was possible to proceed to more approximations to get more precision without it being necessary to change the adjustments. But in most cases the machine was used only for the calculation of

Figure 14. Mallock's calculating machine (Photo C.S.I. Co.)

solutions with large numbers, since the smaller the numbers the less their precision.[61]

An engineer at the Royal Aircraft Establishment in Farnborough tried unsuccessfully to use the machine in 1936–37. During the war, the External Ballistics Department of the Ministry of Supply employed the Mallock machine occasionally when they were using the Mathematical Laboratory, as did people from de Havilland aircraft company, the Royal Establishment, and the National Physical Laboratory.

Croarken concludes that the machine was never really successful even though it was seen as 'a useful and reasonably accurate device.' Aside from some technological deficiencies, it apparently was unable to calculate 'ill-conditioned' equations, which are unavoidable in engineering applications. But there were also some uncontrollable feedback effects which made the machine hawl and pipe. The mathematicians Wilkes and Aitken did not have to deal with any 'ill-conditioned systems' and thus had good results. Presumably all of this would have been true of the Cauer machine, if the prototype had been really completed and used.

There was only a modest interest shown in acquiring a Mallock machine. In 1933 MIT asked, but they were unable to accept the tender of Cambridge Instrument. At the beginning of the war there was an inquiry from General Electric Corporation at Schenectady, and in 1944 another from the Royal Air Force. But because of pressure of competing work the company was not able to respond. In 1944, when Charles Darwin, in his new position as the director of the NPL, suggested the installation of a central calculation office for all governmental institutions, his plan included a Mallock machine. But in 1945 when this proposal became the starting point for the installation of extensive equipment at the NPL Mathematics Division, the Mallock machine was no longer included. After the war, in 1947, the Oscillations Sub-Committee of the Aeronautical Research Council tried to order a machine, but the company was still unable to comply and so gave the patent rights back to Mallock.[62]

Unlike Cauer, Mallock was able to dedicate the greater part of his life to the development of his machine. But because he did so, as Wilkes reports, its lack of commercial success left him a disappointed man. Only the installation of the machine in the Mathematical Laboratory gave him some pleasure. Wilkes noticed that the automatic setting frame was not completely developed in 1937 and concluded that the machine had therefore not been used at the institute at all. Nevertheless the successful pioneer who one decade later created one of the first modern electronic digital computers retrospectively summarized: 'However, it gave me my first introduction to the use of telephone relays in computing, or rather control, circuits and to some of the tricks that one can play with them.'[63]

Some Conclusions

The equation solvers described above are representative of spectrum of machines that were devised during the pre-war years. Although they were commercially unsuccessful, they can be seen as characterizing the historical-technological situation during a critical period, and we can understand the people in the focus of this paper as a community with parallel interests. Each member of this community, responding to common technical problems, was conditioned in his response by a variety of non-technical circumstances. Historians must consider both aspects. It seems that in Cauer's eyes the importance of these machines grew as his chances of becoming a professor declined. As we have seen, his prospects were influenced initially by an academic environment that was inimicable to his mathematical-engineering approach. And increasingly, they were influenced by the political situation in Germany. Mentioned above were Cauer's difficulties with the Nazi party. It is significant that this kind of problem extended even to Courant, who in spite of having been a particularly effective academic representative of the first German republic was despised by the Nazi academics at Göttingen. He had to leave in 1934 despite a new law that said that front-line soldiers from the First World War should be spared.

It is notable that around 1930 this new type of calculating machine for solving linear formulae appeared in two completely different fields of science: engineering, and psychology and social sciences. The former were given special prominence in Europe, the latter in the United States. There was some cross-over, of course. Punched card machines were used in the USA and also in Europe for census work, which can be interpreted as social-scientific problem solving. In 1933 IBM's German daughter Dehomag (Deutsche Hollerith Maschinen AG) offered this machine to the new Nationalsocialist system in Germany to help in effective implementation of its racial politics. For a short time the new situation at Germany seemed to offer a real possibility for empirical social politics on the base of the new technology. But there was no interest in an academic calculating center for social sciences. Instead, Dehomag found its market in the administration and military and industrial bureaucracies.

After 1934 the influence of Vannevar Bush, supported by the large foundations, was overwhelming. Bush not only popularized the technical possibilities of his big mathematical analog machine, he also demonstrated to the world that it could be financed, both in the United States and in other countries with help from the United States. It is difficult to decide if in Göttingen or elsewhere in Europe there was anyone with abilities comparable to those of Bush, but it is certain that nowhere else were there conditions comparable to those that he enjoyed.

In each of the cases presented, at least three concerns had to be addressed. First was to define a problem significantly different from any that which could be accomplished by contemporary desk calculators. Second was to select from numerous possibilities the technical method to be employed. Third was to gain financial support. This last was especially difficult in Europe, while in the United States at least two approaches (Bush and IBM) proved feasible.

There is one more aspect to these pre-computer years: the number of unknowns which the different scientists and engineers felt necessary to be determined was invariably low. One reason might be that they were accustomed to using calculators professionally and did not have experience with as many equations and unknowns as they would need today. Which means that these earlier artefacts might be considered, like telescopes, as capable of revealing a new land (of computing) but at the same time limiting its extent.

Notes

1. W. Aspray, *Computing Before Computers*, Iowa State University Press, Ames, Iowa 1990, 255.
2. Aspray, p. 179.
3. The most important textbook in German is F. A. Willers, *Mathematische Maschinen und Instrumente* (Berlin, 1951). Here one finds 871 cited papers on mathematical instruments and machines. It includes only some of the most important papers in non-German languages. Another textbook is W. Meyer zur Capellen, *Mathematische Instrumente* (Leipzig, 1944), where 310 papers are cited.
4. H. Petzold, 'Maschinen zur Lösung verwickelter mathematischer Probleme. Versuch einer historischen Ortsbestimmung der elektrischen Rechenmaschine Wilhelm Emil Cauers,' in W. Mathis and P. Noll (ed.), *Second ITG-Diskussionssitzung* 21–22, April 1995 (Berlin, 1995), pp. 267–282. This is a preliminary report. I express my thanks to the families of Emil Cauer and Wolfgang Mathis who entrusted Cauer's letters and papers to me, hereafter cited as Cauer-Papers.
5. E. Cauer, 'Familie, Bildung, Wissenschaft. Wilhelm Cauer 1900–1945,' in W. Mathis and P. Noll (ed.), *Second ITG* .... pp. 247–259.
6. As for Hamel, cf. O. Haupt, 'Nachruf auf Georg Hamel,' in *Jahrbuch der Akademie der Wissenschaft*

und Literatur (Wiesbaden, 1954), pp. 148–154, which includes a list of Hamel's publications; L. Prandtl, 'Georg Hamel siebzig Jahre,' in *Zeitschrift für Angewandte Mathematik und Mechanik*, 28 (1948), pp. 129–132; W. Kucharski, *Über Hamels Bedeutung für die Mechanik, in: Zeitschrift für Angewandte Mathematik und Mechanik*, 32 (1952), pp. 293–297. For more on Hamel's mathematical and ideological attitude see G. Hamel, 'Die Bedeutung der Mathematik in der heutigen Zeit,' in *Forschungen und Fortschritte*, 9 (1933), pp. 487–489.

7. The most important biograhical articles in Wilhelm Cauer are: E. Cauer and W. Mathis, 'Wilhelm Cauer (1900–1945),' *Archiv für Elektronik und Übertragungstechnik* 49 (1995), pp. 243–251; and E. Cauer, 'Familie, Bildung, Wissenschaft ....'
8. Hamel's most important books are *Elementare Mechanik* (Leipzig, 1912), and *Theoretische Mechanik* (Berlin, 1949).
9. Only occasionally can historians learn which precise instruments were used to calculate the unknowns of systems of linear equations. In his letter to Cauer from 22.1.1930 Vannevar Bush mentions that his 'ordinary method' of solving systems of linear equations was 'by means of a slide rule of the cylindrical type used together with an adding machine.' Cauer Papers.
10. Twenty-five years later Cauer's doctoral dissertation was accepted within the new engineering discipline of network synthesis. W. Mathis, 'Die Rezeption von Cauers Arbeiten. Aus dem Nachlaß von Wilhelm Cauer,' in *Second ITG ...*, pp. 289–294.
11. The earliest reference I have found is a patent office receipt dated 7 September 1928, Reichspatentamt Berlin. Cauer-Papers.
12. List of Cauer's patents in W. Cauer, *Theorie der linearen Wechselstromschaltungen*, Vol.1, 2nd Edition (Berlin, 1954), p. xvii.
13. W. Cauer, 'Elektrische Maschinen zur Auflösung von Systemen linearer Gleichungen,' *Elektrische Nachrichtentechnik* 12 (1935), 147–157. On page 150 Cauer mentions 'a suggestion' he had made 'already in 1928.'
14. The only description of the device is in W. Cauer, 'Elektrische Maschinen....'
15. C. Reid, *Richard Courant 1888–1972. Der Mathematiker als Zeitgenosse* (Berlin, 1979).
16. Correspondence with Bush between August 1929 and July 1930. Cauer-Papers.
17. W. Cauer, 'Report of my Activities as Fellow of the Rockefeller Foundation,' manuscript without date, obviously end of 1931. Cauer-Papers.
18. N. Wiener, *I am a Mathematician* (London, 1956), p. 142. Wiener wrote Cauer's prename incorrectly as 'Richard' instead of 'Wilhelm'—quite likely a confusion with. As far as I know, in the numerous different editions and translations of Wiener's memoirs this mistake was never corrected. Wiener continued, 'However, the scientist with whom I had the most interesting and profitable contacts was Eberhard Hopf.'
19. Cf. W. Mathis' analysis of the reception of Cauer's work. W. Mathis, 'Die Rezeption von Cauers Arbeiten. Aus dem Nachlaß von Wilhelm Cauer,' *Second ITG...*, 289–294.
20. W. Cauer, *Theorie der linearen Wechselstromschaltungen*, Vol. 1 (Berlin, 1941). Sec. ed. (Berlin, 1954).
21. A critical and conscientious report on the situation at Courant's institute has been written by N. Schappacher, 'Das Mathematische Insititut der Universität Göttingen, 1929–1950,' in H. Becker, et. al., *Die Universität Göttingen unter dem Nationalsozialismus. Das verdrängte Kapitel ihrer 250-jährigen Geschichte* (München, 1987), pp. 345–373.
22. W. Cauer, W., 'Betrifft Auflösung linearer Gleichungen.' Copies of this paper, which is dated from 1 October 1933, were obviously inclosed to Cauer's letters. Cauer Papers. The results of these project were published in: W. Cauer, 'Elektrische Maschinen ....'
23. R. R. Sears, Clark Leonard Hull, in J. A. Garraty (ed.), *Dictionary of American Biography*, suppl. 5 (New York, 1977), pp. 328–331.
24. C. L. Hull, *Aptitude Testing* (Yonkers-on-Hudson, Chicago, 1928). Cf. generally S. J. Gould, *The Mismeasure of Man* (New York, 1996).
25. C. L. Hull, 'An Automatic Correlation Calculating Machine,' *Journal of the American Statistical Association* 20 (1925), 522–531; C. L. Hull, 'An Automatic Machine for Making Multiple Aptitude Forecasts,' *Journal of Educational Psychology* 16 (1925), 593–598; L. L. Thurstone, *Psychological Bulletin* 14 (1917), 28; B. Ruml, *Psychological Bulletin* 13 (1916), 444. An arrangement on Hull's machine gives C. and R. Eames, C. and R., *A Computer Perspective. Background to the Computer Age*, New edition (Cambridge, London, 1990. (first ed. 1973), pp. 70, 71 and 89.
26. C. L. Hull, 'The Joint Yield of Teams of Tests,' *Journal Educational Psychology* 14 (1923), 405.
27. Obviously, this is the price which Cauer referred to.

28. Hull, 'An Automatic Correlation Calculating Machine...,' p. 531.
29. The machine had been received in 1955 with Accession Number 205,424 and Catalog Number 314,605 as 'Calculator, Dr. Clark Hull's coordination (sic!, must be 'correlation') machine.' In the archives of NMAH one can find along with correspondence related to acceptance of the machine a microfilm of several copy-books with Hull's daily notes and sketches of his stepwise design. Clark Hull Papers 1902–51, Reel 3. The original is at the archive of the Yale University. I express my great thanks to Peggy Kidwell for preparing and supporting my inspection of the machine at the depositoty outside of Washington D.C. and also for her reference to the microfilm.
30. I could not find any critical reports looking back upon the work with the machine. Unfortunately I could not investigate exhaustively and generally the history and the importance of Hull's psychological work. So I cannot refer to any critical comments of Hull's psychological fellows.
31. Hull, C. L., *Aptitude Testing*, p. v.
32. Hull, *Aptitude Testing*, pp. 5, 6.
33. Hull, *Aptitude Testing*, pp. 487ff.
34. T. Zech, 'Harmonische Analyse mit Hilfe des Lochkartenverfahrens,' *Zeitschrift für Angewandte Mathematik und Mechanik (ZAMM)* 9 (1929), 425–427.
35. Cauer Papers.
36. W. Rodgers, *Think. A Biography of the Watsons and IBM* (London, 1970), p. 134.
37. Some years later V. Bush became president of the Carnegie Foundation.
38. Rodgers, *Think...* p. 138.
39. E. W. Pugh, *Building IBM. Shaping an Industry and its Technology* (Cambridge, MA, London, 1995), p. 70.
40. Cf. F. W. Kistermann, 'The Way to the First Automatic Sequence-Controlled Caculator: The 1935 DEHOMAG D 11 Tabulator,' *IEEE Annals of the History of Computing* 17 (1995), 33–49, particularly 41.
41. I thank once more Peggy Kidwell for her assistance in my inspection of the machine.
42. Kistermann, 'The Way ....'
43. V. Bush, 'Recent Progress in Analysing Machines,' *Proceedings of the 4th International Congress for Applied Mechanics* (Cambridge, Eng., 1934), 3–23.
44. The Kelvin principle was used practically in numerous tide predicting machines. Cauer doubted still in 1935 that it was possible to control the propagation of the error in a purely digital automatical machine. The only possibility he saw was an analog tunable device. Cauer, 'Elektrische Maschinen..,' p. 154.
45. J. B. Wilbur, 'The Mechanical Solution of Simultaneous Equations,' *Journal of the Franklin Institute* 219 (1936), 715–724.
46. W. W. Leontief, 'Interrelation of Prices, Output, Savings, and Investment,' *The Review of Economic Statistics* 19 (1937), 109–132.
47. Cited in Eames, *A Computer Perespective* ..., p. 113.
48. I thank I. B. Cohen for this information. He had saved Wilbur's machine.
49. V. Bush, 'Structural Analysis by Electric Circuit Analogies,' *Journal of the Franklin Insitute* 217 (1934), 289–329.
50. I am greatly obliged to Mary G. Croarken who intrusted her unpublished manuscript on Mallock and his machine to me and allowed me generously to cite it. M. G. Croarken, 'The Mallock Machine,' unpublished manuscript, August 9, 1984. I am also obliged to Martin Campbell-Kelly who called my attention to the work of Mary Croarken. Cf. also M. G. Croarken, *Early Scientific Computing in Britain* (Oxford, 1990), p. 49 ff.
51. R. R. M. Mallock, 'An Electrical Calculating Machine,' *Proceedings of the Royal Society of London*, Series A, Vol. 140 (1933), 457–483, here 457.
52. As Wilkes reports, in 1937 the machine could solve a system of 10 equations with 10 unknowns. M. V. Wilkes, *Memoirs of a Computer Pioneer* (Cambridge, MA, London, 1985), p. 29.
53. Cited by M. Croarken, 'The Mallock Machine ...,' p. 8.
54. Croarken, 'The Mallock Machine ...,' pp. 7ff.
55. Cf. M. G. Croarken, 'The Emergence of Computing Science Research and Teaching at Cambridge, 1936–1949,' *IEEE Annals of the History of Computing*, Vol. 14, No. 4 (1992), 10–15. Croarken, 'The Mallock Machine,' op. cit., p. 3. The English pioneer of computers, Maurice V. Wilkes, reports in his memoirs how Mallock's machine was bought for the Mathematical Laboratory of the University. The lack of precision, arising from the losses in the transformers, was adjusted by a sophisticated controlling device, the so-called 'compensator' which Wilkes estimated, looking back

with some respect but also a bit relativized, 'As a piece of electronics, this was well ahead of its time.' Wilkes, *Memoirs*, op. cit. P. 29.

56. Cf. M. J. G. Cattermole, A. F. Wolfe, *Horace Darwin's shop. A history of The Cambridge Scientific Instrument Company 1878 to 1968*, (Bristol, Boston, 1987), pp. 136–139. R. R. M. Mallock, 'An Electrical Calculating Machine ....'
57. Anon., 'The Mallock Electrical Calculating Machine,' *Engineering* 137 (1934), 698–700.
58. V. Bush, 'Recent Progress in Analysing Machines ....'
59. Cf. A. C. Aitken, 'Mr. Mallock's Electrical Calculating Machine,' Letter to the editor, February 9, 1935, *Nature* 135 (1935), 235.
60. Croarken, 'Mallock Machine ...,' p. 12.
61. Croarken does not believe the the correctness of some reports which say that the machine had calculated determinants to degree 11 because Mallock nowhere mentioned how it was possible to detect that a determinant was zero.
62. At this time Alain Turing was working at NPL on his computer design and took trouble of its realization. On the other hand, the NPL still some years later gave an offer to the German company Schoppe & Faeser for the design and production of perhaps the biggest large-scale mechanical Differential Analyzer, which was ever built, and which was delivered in 1950. Obviously this machine should never be used. Cf. *Petzold, H.*, Rechnende Maschinen. Eine historische Untersuchung ihrer Herstellung und Anwendung vom Kaiserreich bis zur Bundesrepublik, Düsseldorf 1985, 76–81.
63. Wilkes, 29.